Hardware-in-the-Loop Experiments on Bistability

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SIREN
Stability Islands:
Performance Revolution in Machining
European Research Council

Outline

- **Why Hardware-In-the-Loop (HIL)?**
- Stability islands – can we reach/use them?
- Uncertainties of lobe diagrams: practice & theory
- Testing HIL to reproduce bistable zones: stick-slip
- Hopf bifurcation, stable and unstable limit cycles
- Stability, CM, secondary Hopf bifurcation in HIL
- Bistability in High-Speed-Milling (HSM)
- Experimental setup for HIL in HSM
- Outlook

Why HIL?

Hardware-In-the-Loop (HIL) is an experimental method that accelerates the R&D phase of new machines and technologies

- A certain part of the machine is kept physically, other parts are emulated by digitally controlled actuators based on the simulation of math models
- Similar terminology: sub-structuring
- High Speed Milling (HSM) can use it for near-reality modal tests and/or ***development of new milling tool geometries***
- **But**: extreme challenges occur due to ***high speed***

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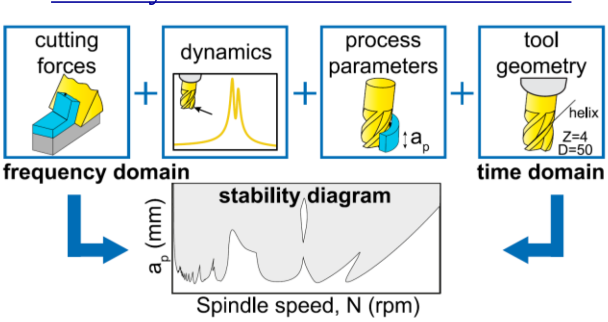
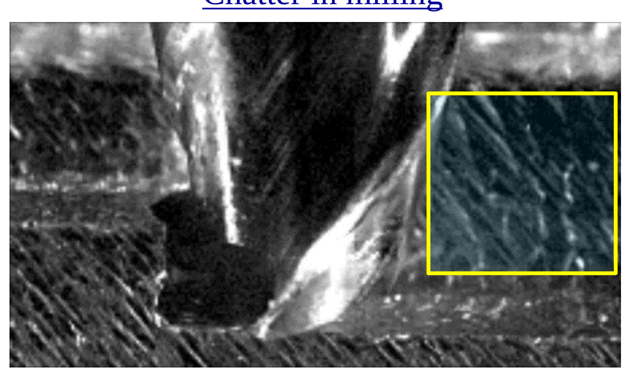
Stability islands – can we reach them?

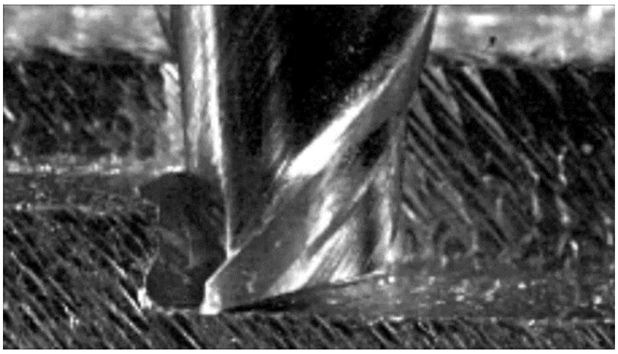


Munoa, …, Stepan, *CIRP Annals* keynote paper on chatter suppression techniques, 2016

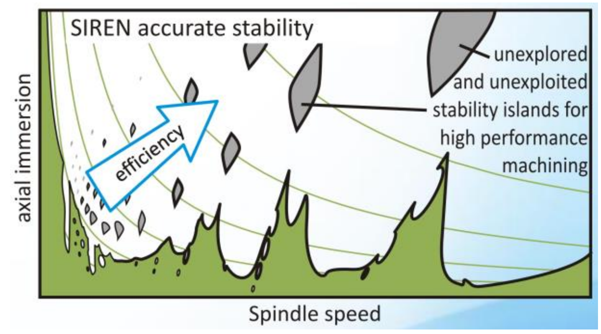
Chatter in milling



Chatter in milling



Stability islands – can we reach them?



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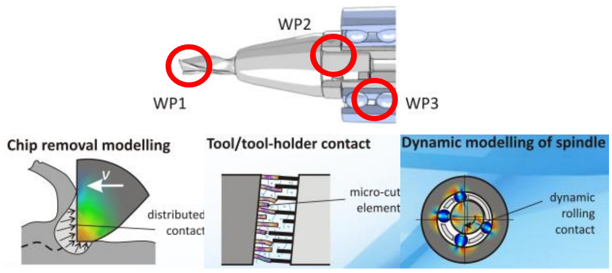
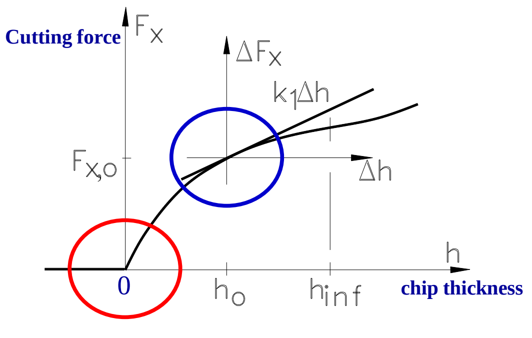
Practice: critical points in modelling



Result: large uncertainties in the lobe structure
high sensitivity for perturbations

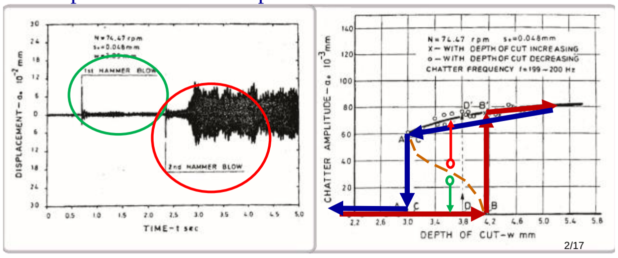
Theory: uncertainty for perturbations

Cutting force nonlinearity against chip thickness

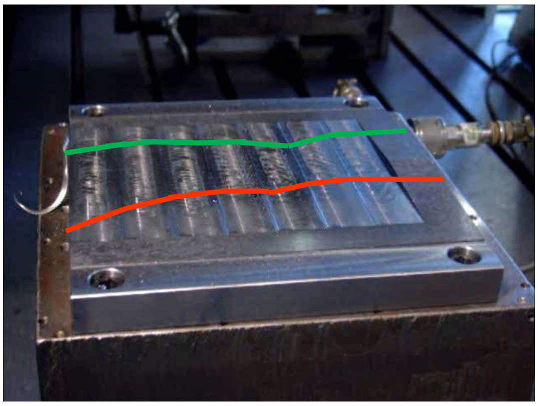


Machine tool vibration – Preliminaries

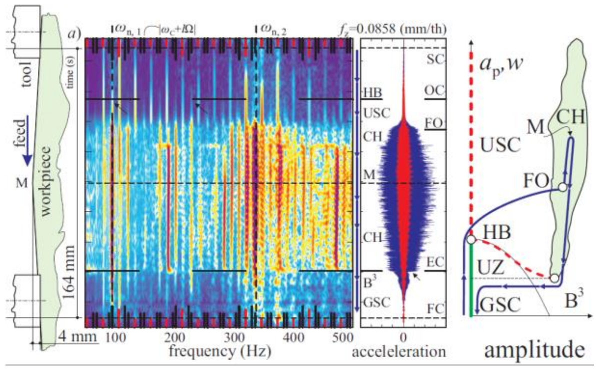
Classical experiment (Tobias, Shi, 1984)
- cutting process is sensitive to large perturbations
- self excited vibrations (chatter) “around” stable cutting
- important effect of chip thickness on size of unsafe zone



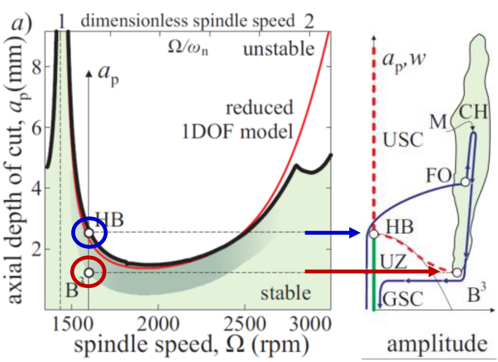
Face milling case study with IDEKO



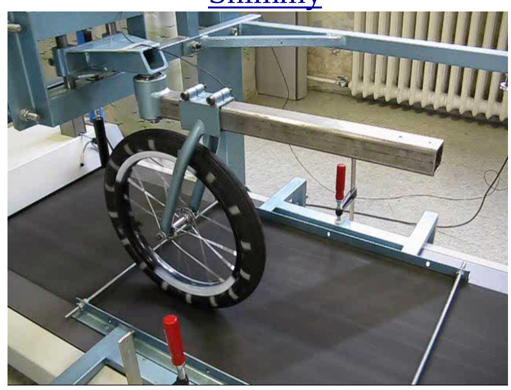
Wavelet, time-history and bifurcations



Stability chart, bistability and bifurcations



Shimmy



Stick-slip

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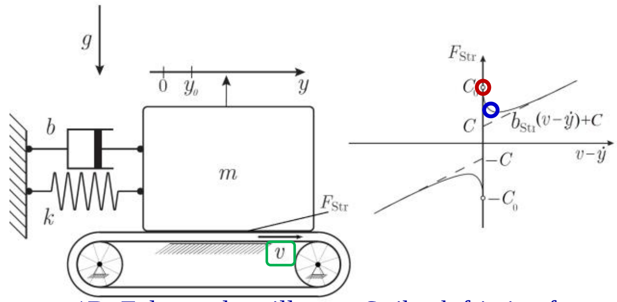
Stick-slip experiment

Simplest nonlinear mechanical model

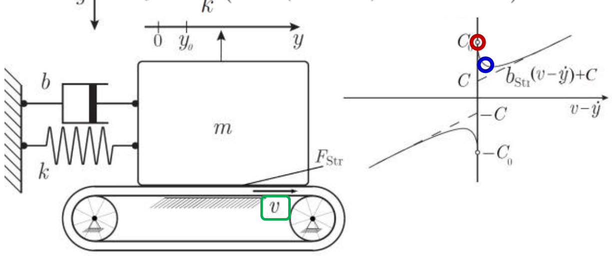


1DoF damped oscillator   Stribeck friction force

see more low DoF models by Leine, Bishop, Dankowitz...
high DoF models by Wiercigroch, Awrejcewicz…

Simplest nonlinear mechanical model

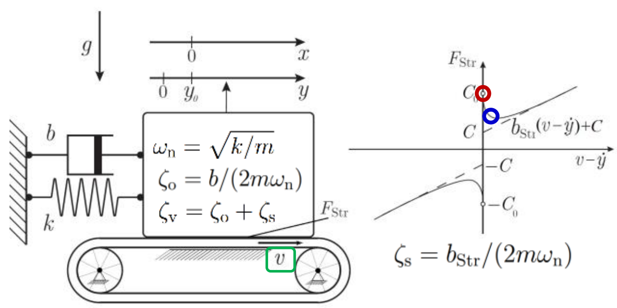
$$y_0 = \frac{1}{k}\left(C + (C_0 - C)\mathrm{e}^{-c_v v} + b_{Str} v\right)$$



$$m\ddot{y} + b\dot{y} + ky = F_{Str}(\dot{y}; v) \qquad \dot{y} \neq v$$

$$F_{Str}(\dot{y}; v) = \left(C + (C_0 - C)\mathrm{e}^{-c_v|v-\dot{y}|}\right)\operatorname{sgn}(v-\dot{y}) + b_{Str}(v-\dot{y})$$

Simplest nonlinear mechanical model

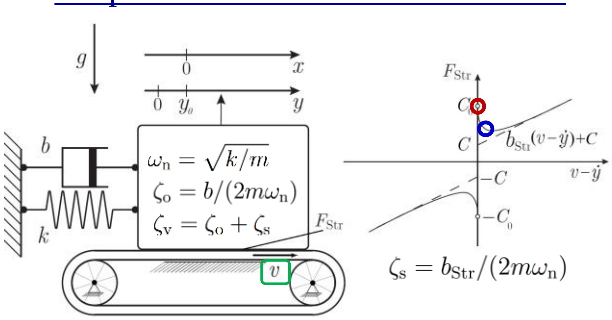


$$\zeta_s = b_{Str}/(2m\omega_n)$$

$$\ddot{x} + 2\zeta_v\omega_n\dot{x} + \omega_n^2 x = \frac{1}{m}(C_0 - C)\mathrm{e}^{-c_v v}(\mathrm{e}^{c_v\dot{x}} - 1)$$

$$\dot{x} < v$$

Simplest nonlinear mechanical model



$$\zeta_s = b_{Str}/(2m\omega_n)$$

$$\ddot{x} + \left(2\zeta_v\omega_n - \frac{C_0 - C}{m\mathrm{e}^{c_v v}}c_v\right)\dot{x} + \omega_n^2 x = \frac{C_0 - C}{m\mathrm{e}^{c_v v}}\left(\frac{c_v^2}{2}\dot{x}^2 + \frac{c_v^3}{6}\dot{x}^3\right)$$

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Stability and Hopf bifurcation

$$\ddot{x} + \left(2\zeta_{\rm v}\omega_{\rm n} - \frac{C_0 - C}{me^{c_{\rm v}v}}c_{\rm v}\right)\dot{x} + \omega_{\rm n}^2 x = \frac{C_0 - C}{me^{c_{\rm v}v}}\left(\frac{c_{\rm v}^2}{2}\dot{x}^2 + \frac{c_{\rm v}^3}{6}\dot{x}^3\right)$$

Equilibrium is stable iff $v > v_{\rm cr,0} := \frac{1}{c_{\rm v}}\ln\left(c_{\rm v}\frac{C_0 - C}{2\zeta_{\rm v}\omega_{\rm n}m}\right)$

Critical char. exponents: $\lambda_{1,2}(v_{\rm cr,0}) = \pm{\rm i}\,\omega_{\rm n}$

Root tendency: $\mathrm{Re}\frac{\mathrm{d}\lambda}{\mathrm{d}v}\Big|_{v=v_{\rm cr,0}} =: \sigma'(v_{\rm cr,0}) = -\zeta_{\rm v}\omega_{\rm n}c_{\rm v} < 0$

Jordan form:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & \omega_{\rm n}\\ -\omega_{\rm n} & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}0\\ (\zeta_{\rm v}\omega_{\rm n}^2 c_{\rm v})x_2^2 + (\zeta_{\rm v}\omega_{\rm n}^3 c_{\rm v}^2/3)x_2^3\end{pmatrix}$$

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$$\delta = \frac{1}{8}\Big(\frac{1}{\omega_{\rm n}}\big((a_{20}+a_{02})(-a_{11}+b_{20}-b_{02}) + (b_{20}+b_{02})(a_{20}-a_{02}+b_{11})\big) + (3a_{30}+a_{12}+b_{21}+3b_{03})\Big)$$

Stability and Hopf bifurcation

$$\ddot{x} + \left(2\zeta_{\rm v}\omega_{\rm n} - \frac{C_0 - C}{me^{c_{\rm v}v}}c_{\rm v}\right)\dot{x} + \omega_{\rm n}^2 x = \frac{C_0 - C}{me^{c_{\rm v}v}}\left(\frac{c_{\rm v}^2}{2}\dot{x}^2 + \frac{c_{\rm v}^3}{6}\dot{x}^3\right)$$

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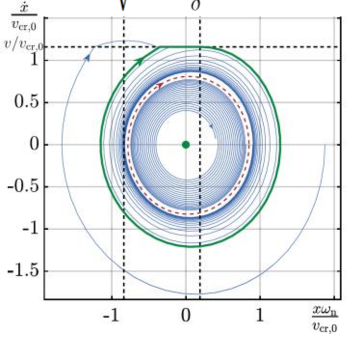
Poincaré-Lyapunov constant:

$\delta = \frac{\zeta_{\rm v}\omega_{\rm n}^3 c_{\rm v}^2}{8} > 0$ ⇨ subcritical Hopf bifurcation, unstable limit cycle for $v > v_{\rm cr,0}$

Unstable and stable limit cycle

$$r(v) \approx \sqrt{-\frac{\sigma'(v_{\rm cr,0})}{\delta}(v - v_{\rm cr,0})} = \frac{2}{\omega_{\rm n}}\sqrt{\frac{2}{c_{\rm v}}(v - v_{\rm cr,0})}$$

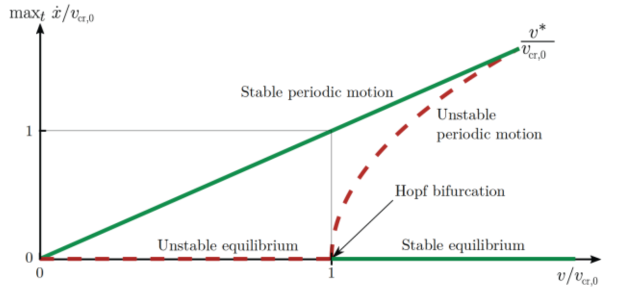


Amplitude r of unstable limit cycle

Stable limit cycle ~ non-uniqueness

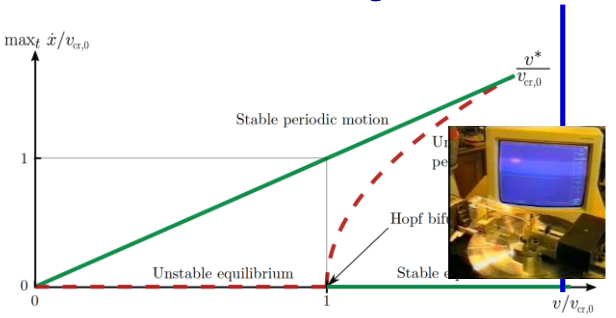
Phase plane

Bifurcation diagram



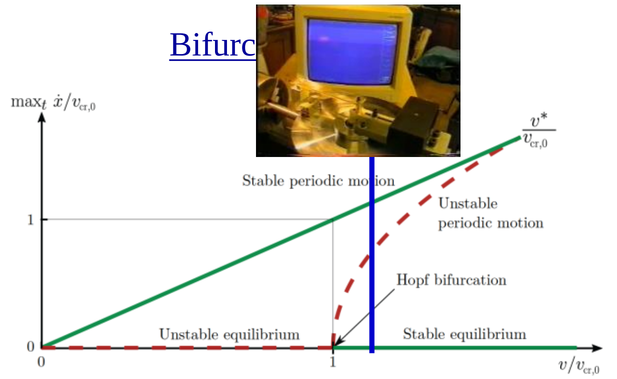
$$v_{\rm cr,0} = \frac{1}{c_{\rm v}}\ln\left(c_{\rm v}\frac{C_0 - C}{2\zeta_{\rm v}\omega_{\rm n}m}\right) \qquad v^* = \frac{4}{c_{\rm v}}\left(1 - \sqrt{1 - c_{\rm v}v_{\rm cr,0}/2}\right)$$

Bifurcation diagram



$$v_{cr,0} = \frac{1}{c_v}\ln\left(c_v\frac{C_0-C}{2\zeta_v\omega_n m}\right) \quad v^\star = \frac{4}{c_v}\left(1-\sqrt{1-c_v v_{cr,0}/2}\right)$$

Bifurc



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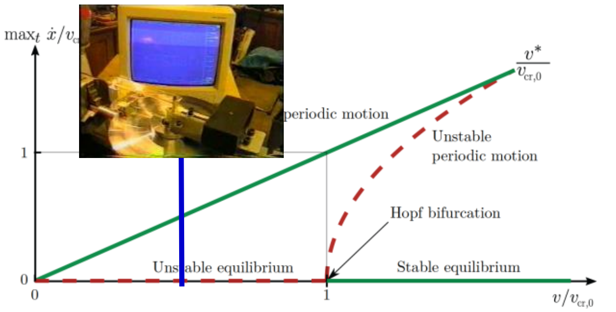
Bifurcation diagram



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Stick-slip at low speed

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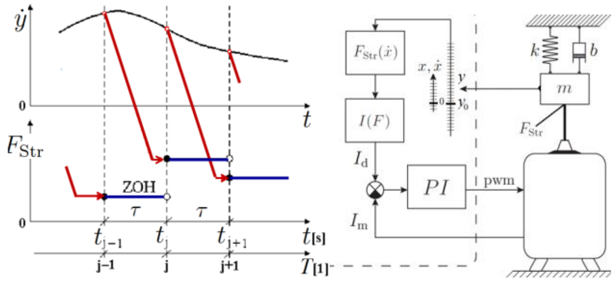
Hardware-in-the-loop experiment



Stribeck force emulated by computer code & control:

$$m\ddot{y}(t)+b\dot{y}(t)+ky(t)=F_{\rm Str}(\dot{y}(t_j-\tau);v)\qquad t_j=j\tau$$
$$t\in[t_j,t_j+\tau)\,,\ j=1,2,\dots$$

Modeling the sampling effect

$$m\ddot{y}(t)+b\dot{y}(t)+ky(t)=F_{\rm Str}(\dot{y}(t_j-\tau);v)\qquad t\in[t_j,t_j+\tau)$$

The equilibrium remains the same in the digital case

$$\ddot{x}(t)+2\zeta_o\omega_n\dot{x}(t)+\omega_n^2x(t)=\frac{C_0-C}{me^{c_v v}}(e^{c_v\dot{x}(t_j-\tau)}-1)-2\zeta_s\omega_n\dot{x}(t_j-\tau)$$

New damping ratios are introduced:

$$\zeta(v)=-\zeta_c(v)+\zeta_s\,,\quad \zeta_c(v)=c_v\frac{C_0-C}{2m\omega_n e^{c_v v}}$$

Dimensionless time: $t=\tau T\,,\quad \dot{x}=\frac{1}{\tau}x'$

Frequency ratio:

$$\omega:=\omega_n\tau=2\pi\frac{f_n}{f_s}$$

f_n ⟸ natural frequency (undamped)

f_s ⟸ sampling frequency

Modeling the sampling effect

$$x''(T)+2\zeta_o\omega x'(T)+\omega^2x(T)=u_j\,,\ T\in[j,j+1)$$

$$u_j=-2\zeta(v)\omega x'(j-1)+\frac{c_v}{\tau}\zeta_c(v)\omega x'^2(j-1)+\frac{1}{3}\left(\frac{c_v}{\tau}\right)^2\zeta_c(v)\omega x'^3(j-1)$$

With the help of the piece-wise closed form solution:

$$\mathbf{x_j}=\begin{pmatrix}x_{1j}\\x_{2j}\\x_{3j}\end{pmatrix}:=\begin{pmatrix}x(j)\\x'(j)\\u_j\end{pmatrix}\qquad \boxed{\mathbf{x_{j+1}}=\mathbf{Q}(v)\,\mathbf{x_j}+\mathbf{f}(\mathbf{x_j};v)}$$

For negligible system damping $\zeta_o=0$

$$\mathbf{Q}(v)=\begin{pmatrix}\cos\omega & \frac{\sin\omega}{\omega} & \frac{1-\cos\omega}{\omega^2}\\ -\omega\sin\omega & \cos\omega & \frac{\sin\omega}{\omega}\\ 0 & -2\zeta(v)\omega & 0\end{pmatrix};\ \mathbf{f}(\mathbf{x_j};v)=\begin{pmatrix}0\\0\\ \frac{c_v}{\tau}\zeta_c(v)\omega x_{2j}^2+\frac{1}{3}\left(\frac{c_v}{\tau}\right)^2\zeta_c(v)\omega x_{2j}^3\end{pmatrix}$$

Stability of the equilibrium

$$\mathbf{x_{j+1}}=\mathbf{Q}(v)\,\mathbf{x_j}+\mathbf{f}(\mathbf{x_j};v);\ \det(\mu\mathbf{I}-\mathbf{Q}(v))=0;\ |\mu_{1,2,3}|<1$$

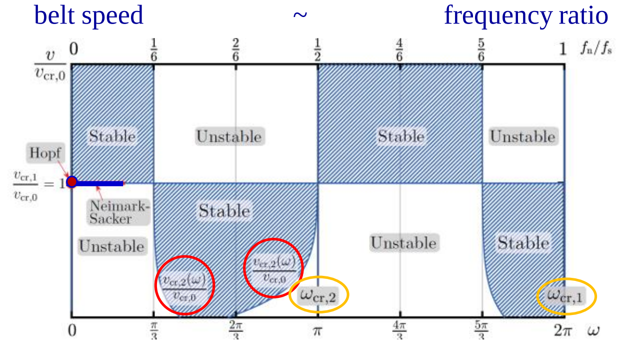
$$\mu^3-2(\cos\omega)\mu^2+(2\zeta(v)\sin\omega+1)\mu-2\zeta(v)\sin\omega=0$$

Critical parameters and characteristic multipliers:

$$\omega_{cr,1}=2j\pi\ \Rightarrow\ \mu_{1,2}=+1\,,\ \mu_3=0;\ (j=1,2,\dots)$$
$$\omega_{cr,2}=(2j-1)\pi\ \Rightarrow\ \mu_{1,2}=-1\,,\ \mu_3=0;\ (j=1,2,\dots)$$
$$\zeta_{cr,1}=0\ \Rightarrow\ \mu_{1,2}=\cos\omega\pm i\sin\omega\,,\ \mu_3=0;$$
$$\zeta_{cr,2}=-\frac{1-2\cos\omega}{2\sin\omega}\ \Rightarrow\ \mu_{1,2}=\frac{1}{2}\pm i\frac{\sqrt{3}}{2}\,,\ \mu_3=-(1-2\cos\omega)$$
$$v_{cr,1}=\frac{1}{c_v}\ln\left(c_v\frac{C_0-C}{2\zeta_s\omega_n m}\right),\quad \text{the same as before!!!}$$
$$v_{cr,2}(\omega)=\frac{1}{c_v}\ln\left(c_v\frac{C_0-C}{(2\zeta_s-(1-2\cos\omega)/\sin\omega)\omega_n m}\right)\ ???$$

Stability chart

belt speed ~ frequency ratio



Neimark-Sacker bifurcation

Equilibrium is stable iff $v > v_{\mathrm{cr},1}$ for $\omega<\pi/3$

Critical char. multipliers: $\mu_{1,2}(v_{\mathrm{cr},1}) = \cos\omega \pm \mathrm{i}\sin\omega$

Root tendency: $\left.\frac{\mathrm{d}|\mu|}{\mathrm{d}v}\right|_{v=v_{\mathrm{cr},1}} = c_{\mathrm{v}}\zeta_{\mathrm{s}}\sin\omega\,(1-2\cos\omega)<0$ for $\omega<\pi/3$

Jordan form:

$$\mathbf{z_{j+1}} = \mathbf{T}^{-1}\mathbf{Q}(v_{\mathrm{cr},1})\mathbf{T}\,\mathbf{z_j} + \mathbf{T}^{-1}\mathbf{f}(\mathbf{T}\mathbf{z_j}; v_{\mathrm{cr},1})$$

$$\begin{pmatrix}\cos\omega & \sin\omega & 0\\ -\sin\omega & \cos\omega & 0\\ 0 & 0 & 0\end{pmatrix} \quad \begin{pmatrix}0\\0\\ c_{020}z_{2j}^2 + c_{011}z_{2j}z_{3j} + c_{030}z_{2j}^3 + \dots\end{pmatrix}$$

Center Manifold reduction: $z_3 = h_{20}z_1^2 + h_{11}z_1z_2 + h_{02}z_2^2$

$$h_{20}=\frac{c_{\mathrm{v}}}{\tau}\zeta_{\mathrm{s}}\omega^3\sin^2\omega,\;\; h_{11}=\frac{c_{\mathrm{v}}}{\tau}\zeta_{\mathrm{s}}\omega^3\sin(2\omega),\;\; h_{02}=\frac{c_{\mathrm{v}}}{\tau}\zeta_{\mathrm{s}}\omega^3\cos^2\omega$$

Restriction to CM:

$$\begin{pmatrix}z_{1,j+1}\\ z_{2,j+1}\end{pmatrix}=\begin{pmatrix}\cos\omega & \sin\omega\\ -\sin\omega & \cos\omega\end{pmatrix}\begin{pmatrix}z_{1,j}\\ z_{2,j}\end{pmatrix}+\begin{pmatrix}\Sigma_{j+k=2,3}\,a_{jk}z_1^jz_2^k\\ \Sigma_{j+k=2,3}\,b_{jk}z_1^jz_2^k\end{pmatrix}$$

Neimark-Sacker bifurcation

$$\begin{pmatrix}z_{1,j+1}\\ z_{2,j+1}\end{pmatrix}=\begin{pmatrix}\cos\omega & \sin\omega\\ -\sin\omega & \cos\omega\end{pmatrix}\begin{pmatrix}z_{1,j}\\ z_{2,j}\end{pmatrix}+\begin{pmatrix}\Sigma_{j+k=2,3}\,a_{jk}z_1^jz_2^k\\ \Sigma_{j+k=2,3}\,b_{jk}z_1^jz_2^k\end{pmatrix}$$

All coefficients are proportional to ζ_{s}^2 except

$$a_{03} = -\frac{1}{3}\left(\frac{c_{\mathrm{v}}}{\tau}\right)^2\zeta_{\mathrm{s}}\omega^2(1-\cos\omega) \quad \text{and} \quad b_{03} = \frac{1}{3}\left(\frac{c_{\mathrm{v}}}{\tau}\right)^2\zeta_{\mathrm{s}}\omega^2\sin\omega$$

Poincaré-Lyapunov constant (see G Orosz, *Physica D*, 2014)

$$\begin{aligned}\Delta = &\frac{1}{8}\Big((3a_{30}+a_{12}+b_{21}+3b_{03})\cos\omega + (3a_{03}+a_{21}-b_{12}-3b_{30})\sin\omega\Big)\\ &+\frac{1}{16}\Big(-(2a_{20}a_{02}+2b_{20}b_{02}+a_{11}^2+b_{11}^2)+2(a_{20}^2+b_{02}^2)(1-2\cos\omega)\cos\omega\\ &-2(a_{02}^2+b_{20}^2)(3-2\cos\omega)(1+\cos\omega)\\ &+(a_{20}b_{11}+a_{11}b_{02})(5+2\cos\omega-4\cos^2\omega)\\ &+(a_{11}b_{20}+a_{02}b_{11})(1+2\cos\omega-4\cos^2\omega)\\ &-\Big((a_{20}+a_{02})(-a_{11}+b_{20}-b_{02})+(b_{20}+b_{02})(a_{20}-a_{02}+b_{11})\Big)\\ &\times\frac{\sin\omega}{1-\cos\omega}(1-6\cos\omega+4\cos^2\omega)\Big)\end{aligned}$$

Neimark-Sacker bifurcation

$$\begin{pmatrix}z_{1,j+1}\\ z_{2,j+1}\end{pmatrix}=\begin{pmatrix}\cos\omega & \sin\omega\\ -\sin\omega & \cos\omega\end{pmatrix}\begin{pmatrix}z_{1,j}\\ z_{2,j}\end{pmatrix}+\begin{pmatrix}\Sigma_{j+k=2,3}\,a_{jk}z_1^jz_2^k\\ \Sigma_{j+k=2,3}\,b_{jk}z_1^jz_2^k\end{pmatrix}$$

All coefficients are proportional to ζ_{s}^2 except

$$a_{03} = -\frac{1}{3}\left(\frac{c_{\mathrm{v}}}{\tau}\right)^2\zeta_{\mathrm{s}}\omega^2(1-\cos\omega) \quad \text{and} \quad b_{03} = \frac{1}{3}\left(\frac{c_{\mathrm{v}}}{\tau}\right)^2\zeta_{\mathrm{s}}\omega^2\sin\omega$$

Poincaré-Lyapunov constant:

$$\Delta \approx \left(\frac{c_{\mathrm{v}}}{\tau}\right)^2\zeta_{\mathrm{s}}\omega^2\sin\omega(2\cos\omega-1)>0 \quad \text{for } 0<\omega\lesssim\pi/3 \Leftrightarrow f_{\mathrm{s}}\gtrsim 6f_{\mathrm{n}}$$

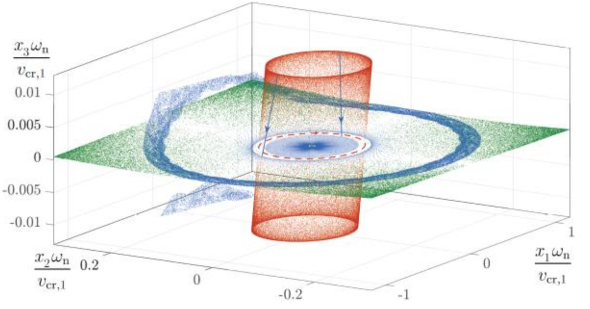
Subcritical NS, unstable periodic iteration

$$r(v)\approx\sqrt{-\frac{|\mu|'(v_{\mathrm{cr},1})}{\Delta}(v-v_{\mathrm{cr},1})}\approx\frac{2}{\omega_{\mathrm{n}}}\sqrt{\frac{2}{c_{\mathrm{v}}}(v-v_{\mathrm{cr},1})}$$

For small system and Stribeck viscous damping and large enough sampling frequency ⇨ same vibration

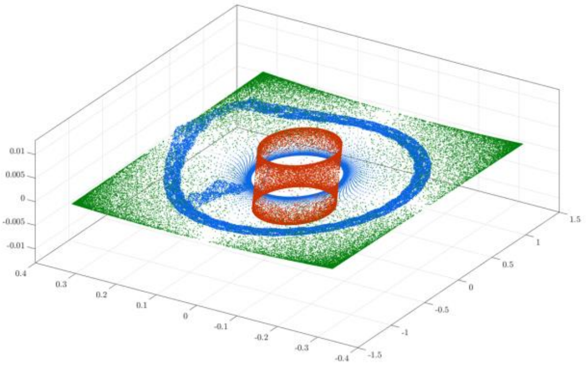
Global dynamics of digital system

Analytical results and numerical simulations



Micro-chaos appears instead of large stable limit cycle due to discretization in space (round-off) (Haller, Stepan)

Global dynamics of digital system

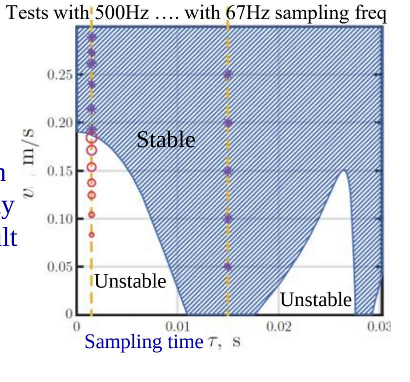


Stability chart with (slight) system damping

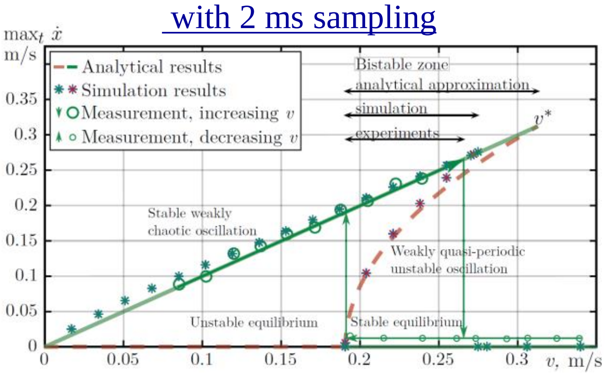
Sampling time of digital control is critical:

2 ms good match

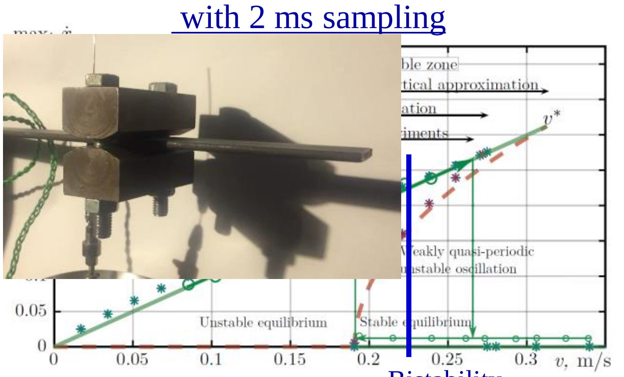
15 ms qualitatively wrong result



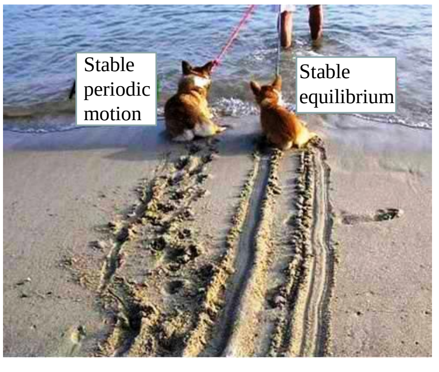
Experimental bifurcation diagram with 2 ms sampling



Experimental bifurcation diagram with 2 ms sampling



Bistability



Outline

- Why Hardware-in-the-Loop (HIL) for HSM?
- Stability islands – can we reach/use them?
- Uncertainties of lobe diagrams: practice & theory
- Testing HIL to reproduce bistable zones: stick-slip
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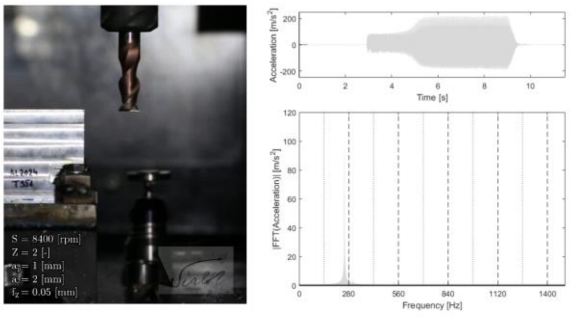
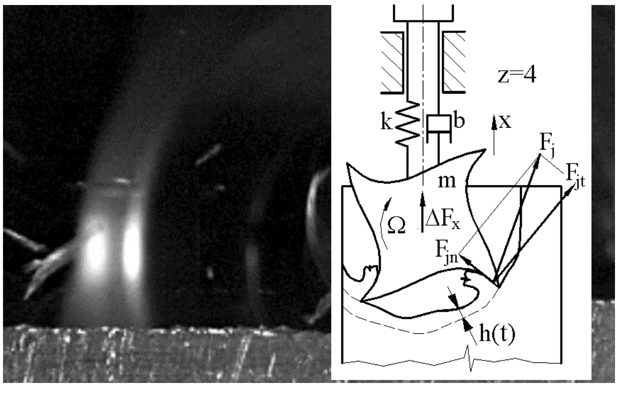
- **Bistability in High-Speed-Milling (HSM)**
- Experimental setup for HIL in HSM
- Outlook

Introduction to milling



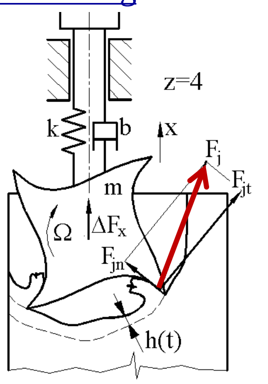
Introduction to milling



Introduction to milling

Number of cutting edges in contact varies ***periodically*** with period equal to the delay between two subsequent cutting edges.

Thus, the resultant cutting force also varies with the same period.



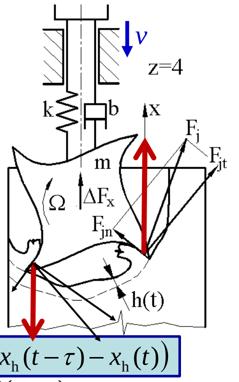
Introduction to milling

Model of milling

Mechanical model:

- number of cutting edges in contact, and
- *x* components of the cutting forces

vary *periodically* with period equal to the delay $\tau = (2\pi/\Omega)/z$



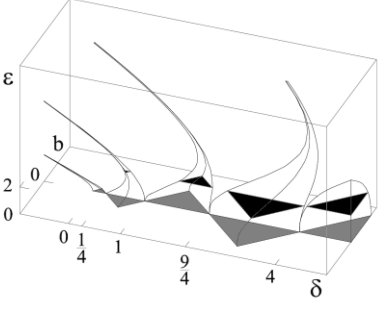
$$m\ddot{x}_h(t) + b\dot{x}_h(t) + kx_h(t) = G(t)\big(x_h(t-\tau) - x_h(t)\big)$$

$$G(t) = G(t+\tau)$$

$$x(t) = x_p(t) + x_h(t), \quad x_p(t) = x_p(t+\tau)$$

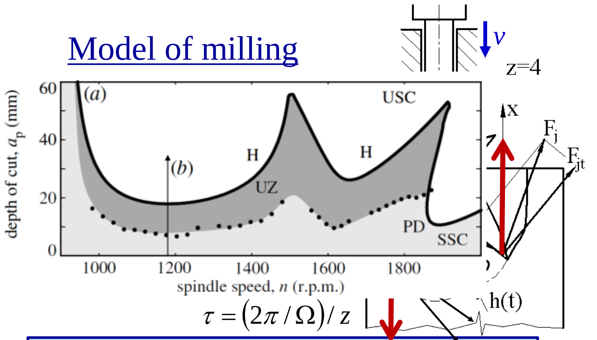
Stability chart of delayed Mathieu

$$\ddot{x}(t) + (\delta + \varepsilon\cos t)x(t) = b\,x(t-2\pi)$$



Insperger, Stepan
Proc. Roy. Soc. A
(2002)

Model of milling



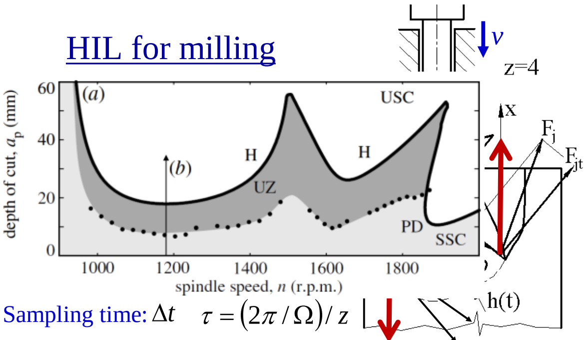
$\tau = (2\pi/\Omega)/z$

$$m\ddot{x}_h(t) + b\dot{x}_h(t) + kx_h(t) = G(t)\big(x_h(t-\tau) - x_h(t)\big)$$

$$G(t) = G(t+\tau)$$

$$x(t) = x_p(t) + x_h(t), \quad x_p(t) = x_p(t+\tau)$$

HIL for milling



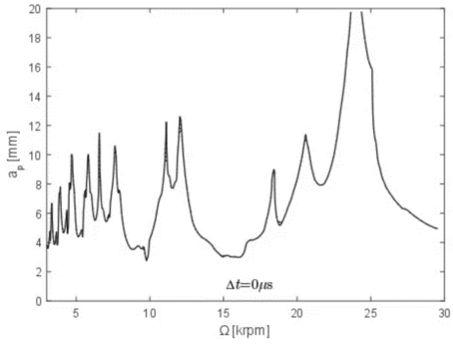
Sampling time: Δt $\quad \tau = (2\pi/\Omega)/z$

$$m\ddot{x}_h(t) + b\dot{x}_h(t) + kx_h(t) = G(t)\big(x_h(t-\tau) - x_h(t)\big)$$

$$m\ddot{x}_h(t) + b\dot{x}_h(t) + kx_h(t) = G(k\Delta t) \times \big(x_h((k-r)\Delta t) - x_h((k-1)\Delta t)\big) \quad t \in [k\Delta t, (k+1)\Delta t)$$

HIL for High-Speed-Milling

Stability chart with increasing sampling time Δt



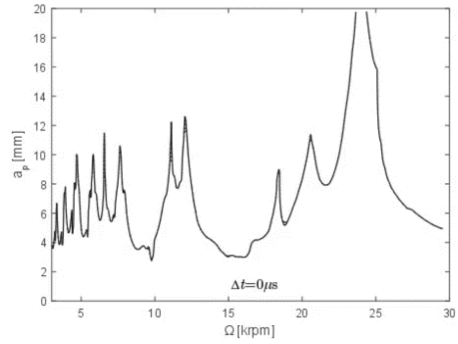
$\omega_n/(2\pi) =$ 785 Hz

$z = 2$

$\tau = 1...10$ ms

HIL for High-Speed-Milling

Stability chart with increasing sampling time Δt



$\omega_n/(2\pi) =$ 785 Hz

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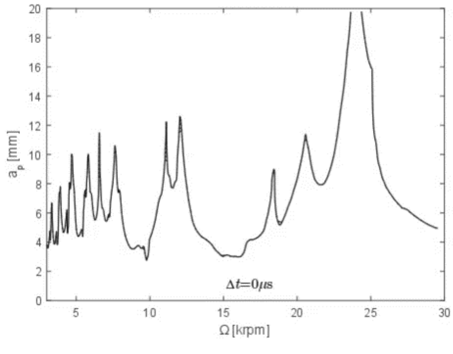
HIL for High-Speed-Milling

Stability chart with increasing sampling time Δt



$\omega_n/(2\pi) =$ 785 Hz

$z = 2$

$\tau = 1...10$ ms

Outline

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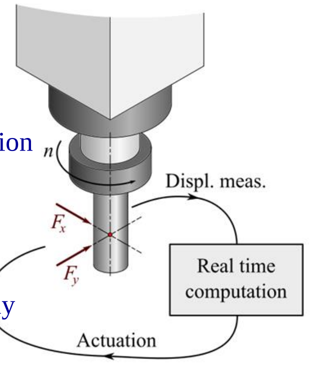
HIL: semi-virtual manufacturing

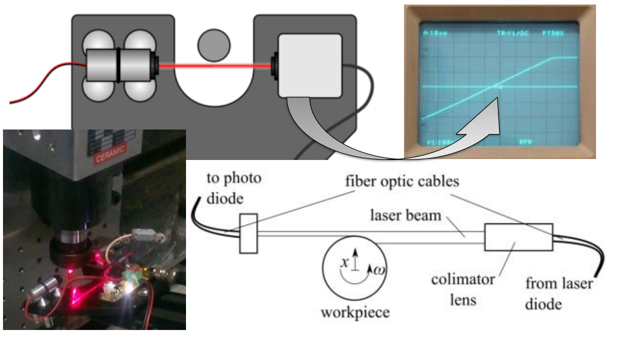
Requirements:

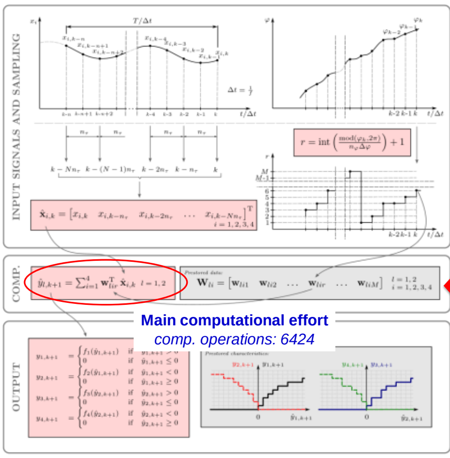
sensor accuracy < 0.1μm
  contactless, direct
  displacement, no integration
sampling freq.=100kHz
  at 30 000 rpm:
  200 samples/rotation
force magnitude >50N
  steady-state force can only
  be partially reproduced



Sensing position on rotating spindle

Photonic beam reduction (FBR) sensor
range: 0.2mm, bandwidth 500kHz, noise < 10nm



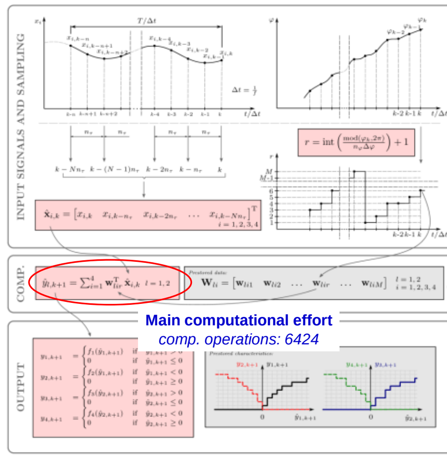


Digital control

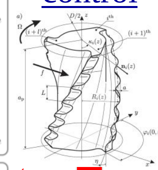
Tool geometry, machining process parameters

Matrix size: 512x400

rotational res.: 512



Digital control



Matrix size: 512x400

rotational res.: 512

Digital control

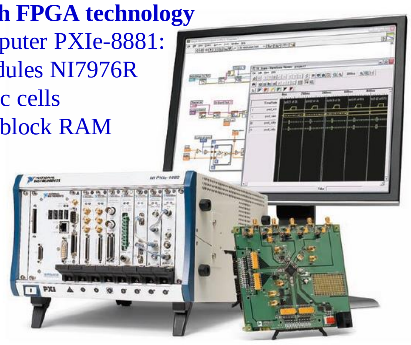
Performed with FPGA technology

Real target computer PXIe-8881:

2 FPGA modules NI7976R

406 720 logic cells

28 620 Kbit block RAM



Actuator at rotating tool

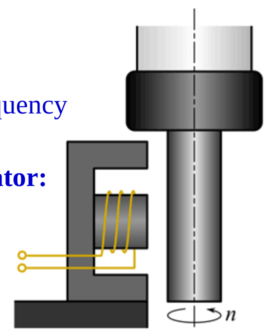
Requirements:

- contactless actuation
- 100 kHz sampling frequency
- appr. 15 N force

Electro magnetic actuator:

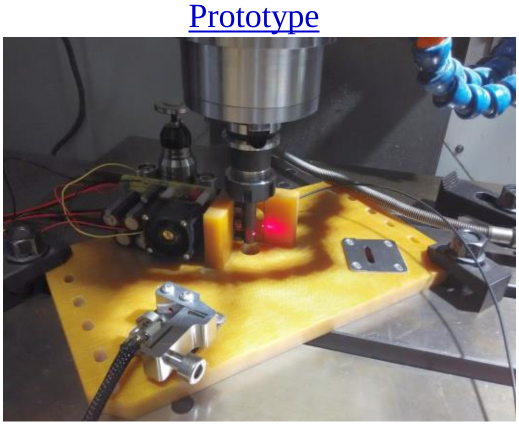
Problems:

- eddy current
- heat transfer
- current control



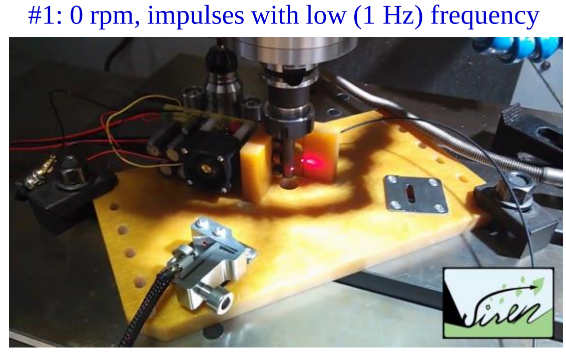
(Rantatalo *et al*. 2007, Yamazaki *et al*. 2010, Matsubara *et al*. 2015)

Prototype

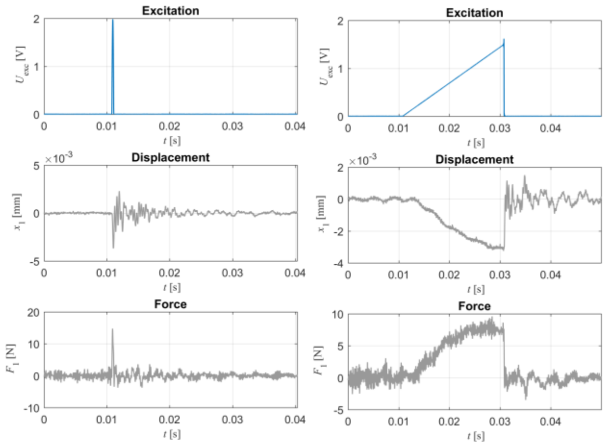


Series of tests

#1: 0 rpm, impulses with low (1 Hz) frequency

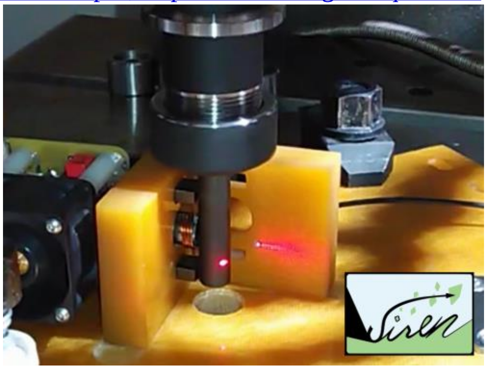


1000 spikes each having 30μs width result in 0.1s non-zero force



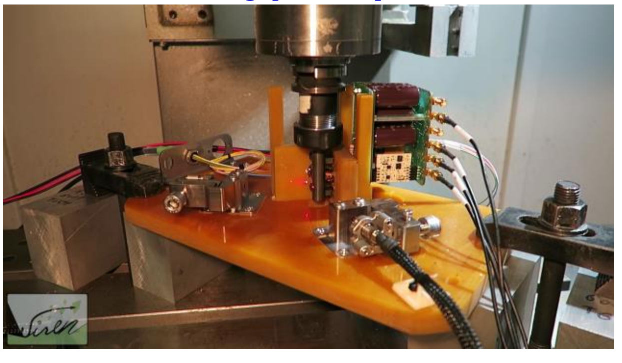
Series of tests

#2: 600 rpm, impulses with high frequencies

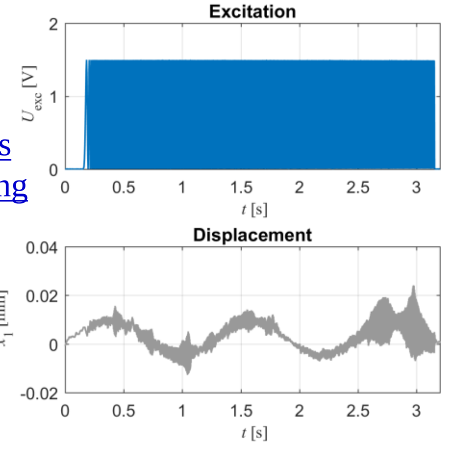


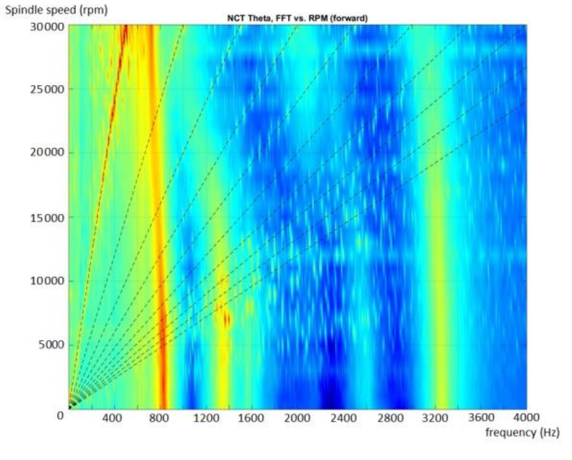
Series of tests

#3: increasing rpm, sweep excitation



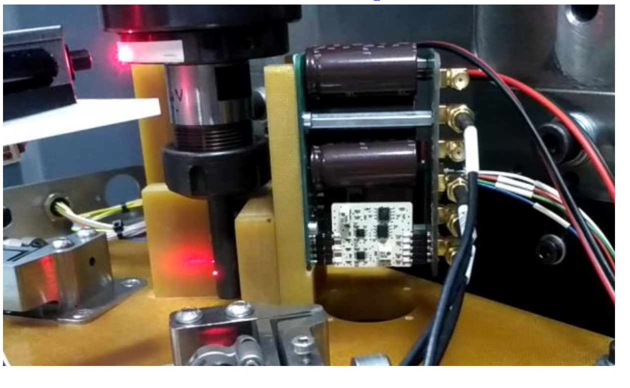
Modal tests with rotating spindle





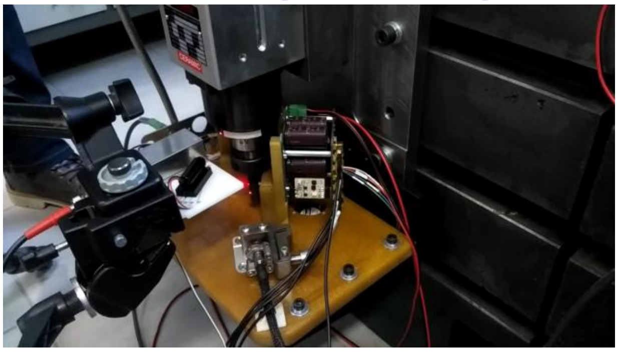
Series of tests

#4: Close the loop !!!



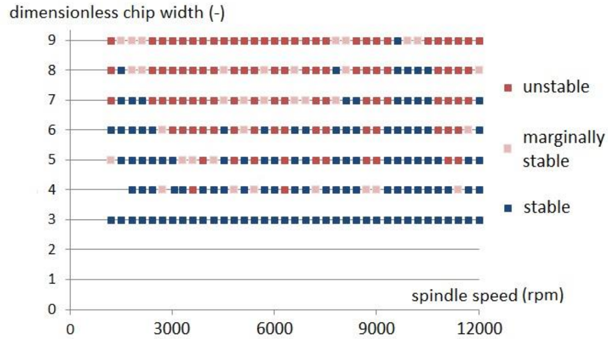
Series of tests

#5: Instabilities at real speeds & virtual depths of cut



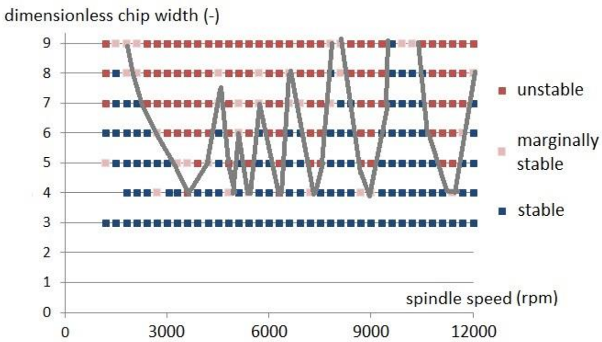
Series of tests

#6: Measure semi-virtual stability chart



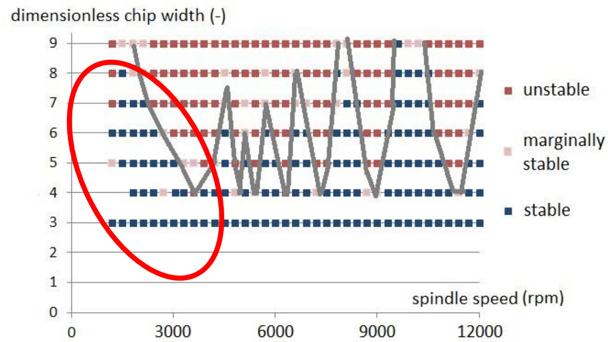
Series of tests

#6: Construct semi-virtual stability chart



Series of tests

#6: This is not process damping (unfortunately…)



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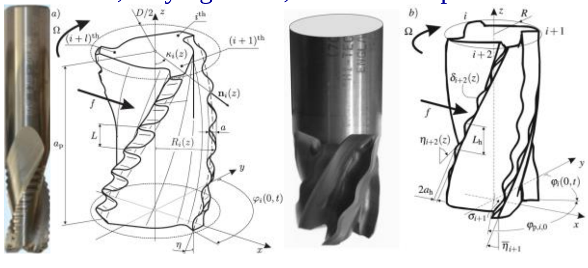
- **Outlook**

Outlook

Serrated-, varying helix-, non-uniform pitch tools…

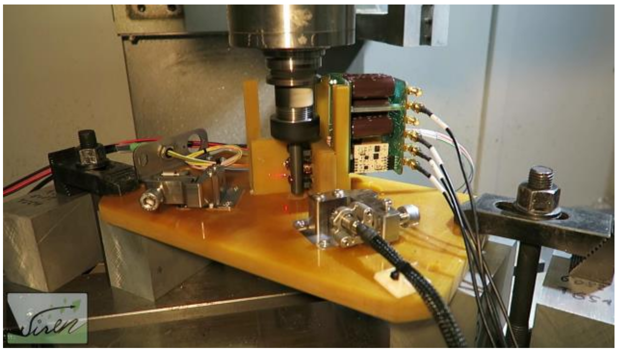


… with varying spindle speed, active vibration absorbers – the models may/will fail, too complex

HIL: Semi-virtual high-speed milling

- Improved modal test ("almost" real conditions and real time)
- Development of special tool geometries without prototyping
- Test manufacturing without material consumption
- Identify the unknown effects and their sources in milling process (with known cutting force)
- Test theoretical spindle and bearing models
- Compare and verify cutting force theories
- Identify theoretically stable parameter domains

EU project / Beethoven's 9th symphony



Thank you for your attention!